

A CLOSE CORRELATION BETWEEN THIRD KEPLER LAW AND TITIUS-BODE RULE

Vladan Panković^{*,‡}, Aleksandar-Meda Radaković[‡]

^{*}Department of Physics, Faculty of Sciences, 21000 Novi Sad,
Trg Dositeja Obradovića 4. , Serbia, vdpan@neobee.net

[‡]Gimnazija, 22320 Indjija, Trg Slobode 2a, Serbia

PACS number: 96.35.-j

Abstract

In this work we present a close correlation between third Kepler law and Titius-Bode empirical rule. Concretely, we demonstrate that third Kepler law, or, corresponding equilibrium condition between centrifugal and Newtonian gravitational force, implies that planet orbital momentum becomes effectively a function of the planet distance as unique variable and vice versa. Then, approximation of the planet distance by its first order Taylor expansion over planet orbital momentum holds an exponential form corresponding to Titius-Bode rule. In this way it is not necessary postulate exponential form of the planet distance (as it has been done by Scardigli) but only discrete values of its argument. Physically, it simply means that, in the linear approximation, "quantized" planets orbital momentums do a geometrical progression.

In this work we shall present a close correlation between third Kepler law and Titius-Bode empirical rule [1], [2]. Concretely, we shall demonstrate that third Kepler law, or, corresponding equilibrium condition between centrifugal and Newtonian gravitational force, implies that planet orbital momentum becomes effectively a function of the planet distance as unique variable and vice versa. Then, approximation of the planet distance by its first order Taylor expansion over planet orbital momentum holds an exponential form corresponding to Titius-Bode rule. In this way it is not necessary postulate exponential form of the planet distance (as it has been done by Scardigli [1]) but only discrete values of its argument. Physically, it simply means that, in the linear approximation, "quantized" planets orbital momentums do a geometrical progression.

Consider well-known situation when a relatively small physical system, e.g. a planet in Sun system, stably rotates, by means of Newtonian gravitational force, along a circumference about central, massive system, e.g. Sun. It corresponds to equilibrium between centrifugal and Newtonian gravitational force, i.e. to expression

$$\frac{mv^2}{R} = \frac{GmM}{R^2}. \quad (1)$$

Here m represents the planet mass, M - Sun mass, R - planet orbit radius or distance between planet and Sun, $v = \frac{2\pi R}{T}$ - planet speed, T - revolution period and G - Newtonian gravitational constant.

After division of (1) by $\frac{m}{R}$ and use of the definition of v as $\frac{2\pi R}{T}$, it simply follows

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM} \quad (2)$$

that represents the remarkable third Kepler law.

Expression (1) represents generally speaking a functional dependence between two variables, v and R . Value of one of these variables, e.g. v , can be chosen, in principle, quite arbitrarily and then, according to (1), there is a practically continuous spectrum of the values of other variable, e.g. R .

But, as it is well-known too, according to famous Titius-Bode [1] empirical rule in Richardson form [1], [2]

$$R_n = R_1 \exp[2\alpha(n - 1)] \quad \text{for } n = 1, 2, 3, \dots \quad (3)$$

it seems that planets orbits are discretized, i.e. "quantized", where R_1 represents Mercury orbit radius, R_2 - Venus orbit radius, etc. and $2\alpha = 0.53707$ corresponding parameter characteristic for Sun system. If Titius-Bode rule does not represent a coincidence only, its existence implies an additional, "quantization" physical condition (dynamical or kinematical) *strongly* different from equilibrium condition (1) or Kepler law (2), as it is presented in [1], [2] etc.

For example Scardigli [2], in an incomplete analogy with Bohr momentum quantization, postulated

$$J = mvR = mS \exp[\alpha n] \quad \text{for } n = 1, 2, 3, \dots \quad (4)$$

where $J = mvR$ represents a planet orbital momentum and S - an additional parameter. Then (1) and (4) represent the equations system with two variables R and v , whose unique solution predict "quantized" form of R equivalent to (3). Explanation of this postulate Scardigli gives by introduction of a more complex theory, i.e. a more accurate, Schrödinger-like dynamics of the global structure of planetary system.

It can be observed that condition (1) can be simply, i.e. by multiplication with mR^3 , transformed in the following expression

$$J^2 = m^2 v^2 R^2 = (Gm^2 M)R \quad (5)$$

that implies

$$R = (Gm^2 M)^{-1} J^2. \quad (6)$$

Expressions (5) and (6) are very interesting. Namely, according to its definition, $J = mvR$, planet orbital momentum J represents the function of two practically independent variables, planet circumference radius R , and planet speed v . However, according to equilibrium condition (1), J becomes function of only one variable R (5), and vice versa (6).

Now, we shall approximate (6) by its first order Taylor expansion within a small vicinity $\Delta J = J - J_0$ of an orbital momentum value J_0 . It yields

$$\begin{aligned} R &\simeq (Gm^2 M)^{-1} J_0^2 + 2(Gm^2 M)^{-1} J_0 \Delta J = \\ &= (Gm^2 M)^{-1} J_0^2 + 2(Gm^2 M)^{-1} J_0^2 \frac{\Delta J}{J_0} = R_0 + 2R_0 \frac{\Delta J}{J_0} = R_0(1 + 2\frac{\Delta J}{J_0}) \end{aligned} \quad (7)$$

where $R_0 = (Gm^2 M)^{-1} J_0^2$. Since $(1 + 2\frac{\Delta J}{J_0})$ can be considered as first order Taylor expansion of the expression $\exp[2\frac{\Delta J}{J_0}]$ we shall suggest a more accurate expression

$$R = R_0 \exp[2\frac{\Delta J}{J_0}]. \quad (8)$$

It and (1) imply

$$v = v_0 \exp\left[-\frac{\Delta J}{J_0}\right] \quad (9)$$

where $v_0 = (GM)^{\frac{1}{2}}R_0^{-\frac{1}{2}}$. Also, according to definition of J and (8), (9), it follows formally

$$J = J_0 \exp\left[\frac{\Delta J}{J_0}\right] \quad (10)$$

that is satisfied in the first order approximation, i.e. Taylor expansion, where $J_0 = mv_0R_0$. Suppose now that there is a discrete planet orbital momentum series corresponding to (10)

$$J_n = J_{n-1} \exp\left[\frac{\Delta J_n}{J_{n-1}}\right] \quad \text{for } n = 2, 3, \dots \quad (11)$$

It, obviously, can be transformed in

$$J_n = J_1 \exp\left[\frac{\Delta J_n}{J_{n-1}} + \frac{\Delta J_{n-1}}{J_{n-2}} + \dots + \frac{\Delta J_2}{J_1}\right] \quad \text{for } n = 1, 2, 3, \dots \quad (12)$$

Suppose additionally

$$\frac{\Delta J_n}{J_{n-1}} = \alpha \simeq \text{const} \quad \text{for } n = 2, 3, \dots \quad (13)$$

where $\Delta J_n = J_n - J_{n-1}$ for $n = 2, 3, \dots$. It means that given planet orbital momentum series J_1, J_2, \dots, J_n represents a geometrical progression with coefficient $1 + \alpha$. Then (12) turns out in

$$J_n = J_1 \exp[(n-1)\alpha] \quad \text{for } n = 2, 3, \dots \quad (14)$$

It implies

$$R_n = R_1 \exp[2(n-1)\alpha] \quad \text{for } n = 2, 3, \dots \quad (15)$$

and

$$v_n = v_1 \exp[-(n-1)\alpha] \quad \text{for } n = 2, 3, \dots \quad (16)$$

Obviously, (15) has the form equivalent to Richardson form of Titius-Bode empirical rule (3). In this way it is proved that Titius-Bode rule follows directly from equilibrium condition (1) or third Kepler law (2) under an additional, weak, "quantization" physical condition (13) which, physically, simply means that, in the linear approximation, "quantized" planets orbital momentums do a geometrical progression. (Given condition is weak in the sense that it is not necessary postulate exponential form of R but only discrete values of the argument of exponential function.)

References

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- [2] F. Scardigli, *A Quantum-like Description of the Planetary Systems*, gr-qc/0507046, and references therein